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Hence,

$$\begin{aligned}\int_a^b \log x\,dx &= \lim_{q\doteq 1}\,[a(q-1)\log a + aq(q-1)\log aq + \cdots + aq^{n-1}(q-1)\log aq^{n-1}],\\ &= \lim_{q\doteq 1} a(q-1)[(1+q+\cdots+q^{n-1})\log a + (q+2q^2+\cdots+(n-1)q^{n-1})\log q],\\ &= \lim_{q\doteq 1} a(q-1)\left[\left(\frac{q^n-1}{q-1}\right)\log a + \left\{\frac{q-nq^n+(n-1)q^{n+1}}{(1-q)^2}\right\}\log q\right],^*\\ &= \lim_{q\doteq 1} a\left[(q^n-1)\log a + \frac{q(1-q^n)}{q-1}\log q + nq^n\log q\right],\\ &= \lim_{q\doteq 1} a\left[\left(\frac{b-a}{a}\right)\log a + \frac{q}{q-1}\left(\frac{a-b}{a}\right)\log q + \frac{b}{a}\log\frac{b}{a}\right],\\ &= b\log b - a\log a + b - a.\end{aligned}$$

Also solved similarly by P. PENALVER.

MECHANICS.

292. Proposed by C. N. SCHMALL, New York City.

In a bombardment, a battleship directs its fire at a fort standing on a hill whose height is a feet above sea level. The angle of elevation of the fort is found to be ϕ. If the initial velocity of the projectile is v, show that the fort will *not* be struck if $v < \sqrt{ag(1+\csc\phi)}$.

SOLUTION BY PAUL CAPRON, Annapolis, Maryland.

If the projectile is fired at an elevation θ, its trajectory may be represented by $x = vt\cos\theta$, $y = vt\sin\theta - \frac{1}{2}gt^2$, or by

$$y = x\tan\theta - \frac{gx^2}{2v^2}\sec^2\theta.$$

To find the envelope of all such trajectories, we have

$$0 = x\sec^2\theta - \frac{gx^2}{v^2}\sec^2\theta\tan\theta, \qquad \text{or} \qquad \tan\theta = \frac{v^2}{gx};$$

whence

$$y = \frac{v^2}{2g} - \frac{gx^2}{2v^2}$$

is the envelope.

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[*] To sum

$$\begin{aligned} q+2q^2+3q^3+\cdots+(n-1)q^{n-1} &= q(1+2q+3q^2+\cdots+(n-1)q^{n-2}) = q\Sigma\\ \Sigma &= 1+2q+3q^2+\cdots+(n-1)q^{n-2}\\ -q\Sigma &= -q-2q^2-\cdots-(n-2)q^{n-2}-(n-1)q^{n-1}\\ \hline \therefore\ (1-q)\Sigma &= 1+q+q^2+\cdots+q^{n-2}-(n-1)q^{n-1}\\ &= \frac{q^{n-1}-1}{q-1}-(n-1)q^{n-1}\\ \therefore\ q\Sigma &= \frac{q-nq^n+(n-1)q^{n+1}}{(1-q)^2}. \end{aligned}$$

The condition that the fort shall not be hit is given by

$$x = a \cot \phi, \qquad a > y;$$

or by

$$a > \frac{v^2}{2g} - \frac{ga^2 \cot^2 \phi}{2v^2} \qquad \text{or} \qquad v^4 - 2agv^2 < a^2g^2 \cot^2 \phi,$$

or

$$(v^2 - ag)^2 < a^2g^2 \csc^2 \phi, \qquad \text{or} \qquad v < \sqrt{ag(1 + \csc \phi)}.$$

Also solved by L. Sivian, F. C. Feemster, A. M. Harding, and Clifford N. Mills.

NUMBER THEORY.

220. Proposed by E. T. BELL, Seattle, Washington.

Let $[m/n]$ denote the greatest integer that is not greater than m/n; and let the two sets,

$$\left[\frac{m}{m-1}\right]; \left[\frac{m+1}{m-2}\right]; \left[\frac{m+2}{m-3}\right]; \cdots \left[\frac{2m-3}{2}\right],$$

$$\left[\frac{m-1}{m-1}\right]; \left[\frac{m}{m-2}\right]; \left[\frac{m+1}{m-3}\right]; \cdots \left[\frac{2m-4}{2}\right],$$

be denoted by (A) and (B) respectively.

Prove that a necessary and sufficient condition that $2m - 1$ be a prime number is that the excess of the number of even integers in (A) over the number of even integers in (B) shall be equal to the excess of the number of odd integers in (A) over the number of odd integers in (B).

Solution by Elijah Swift, University of Vermont.

Unless one of the quantities $m/(m-1)$, $(m+1)/(m-2)$, $\cdots$, $(2m-3)/2$ is an integer the two sets (A) and (B) will be exactly the same. If one of the above quantities is an integer we shall show (1) that it is an even integer; and (2) that $2m - 1$ will then possess a factor.

Let $(m + k)/(m - k - 1)$ = an integer, say α. This is equivalent to the equation $2m - 1 = (\alpha + 1)(m - k - 1)$, which proves the statements above.

Suppose now that $2m - 1$ is prime. Then sets (A) and (B) will be exactly the same. Hence the condition is necessary. If $2m - 1$ is composite, each factor is odd. Call them $\alpha + 1$ and $m - k - 1$, k must be as large as 1. But $2m - 1 = (\alpha + 1)(m - k - 1)$ is the same as $(m + k)/(m - k - 1) = \alpha =$ an integer. Hence, at least one of the integers of (A) is even and the condition can not be fulfilled. Hence the condition is sufficient.